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Research Article

Output-Feedback Controller Based Projective Lag-Synchronization of Uncertain Chaotic Systems in the Presence of Input Nonlinearities

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This paper solves the problem of projective lag-synchronization based on output-feedback control for chaotic drive-response systems with input dead-zone and sector nonlinearities. This class of the drive-response systems is assumed in Brunovsky form but with unavailable states and unknown dynamics. To effectively deal with both dead-zone and sector nonlinearities, the proposed controller is designed in a variable-structure framework. To online learn the uncertain dynamics, adaptive fuzzy systems are used. And to estimate the unavailable states, a simple synchronization error is constructed. To prove the stability of the overall closed-loop system (controller, observer, and drive-response system) and to design the adaptation laws, a Lyapunov theory and strictly positive real (SPR) approach are exploited. Finally, three academic examples are given to show the effectiveness of this proposed lag-synchronization scheme.

1. Introduction

The chaos synchronization has attracted great attention and has been extensively studied [1–14], since it was suggested originally by Pecora and Carroll in [15]. The basic configuration of chaos synchronization consists of two chaotic systems: a drive (master) system and a response (slave) system. These systems can be identical but with different initial conditions (IC) or quite different. The response system is driven via some transmitted (drive) signals so that the trajectories of the response system synchronize with that of the drive system.

In the literature, there are many types of the chaos synchronization such as complete synchronization (CS) [1, 2], generalized synchronization (GS) [3, 4], projective synchronization (PS) [5, 6], and lag-synchronization (LS) [7]. In PS, the state vectors of two synchronized systems evolve in a proportional scale. In LS, due to signal propagation delays in the environment, it is reasonable to require the response system at time (t) to synchronize the drive one at time $(t-\tau)$, where τ is the propagation delay (lag) [16]. In recent years, lag-synchronization has attracted a great deal of attention. Some results have been reported about LS [8–14, 16–18]. Besides, over the past 25 years, a variety of methods have been proposed for chaos synchronization, such as sliding mode control [19, 20], active control [21, 22], adaptive control [23, 24], and fuzzy control [25–27] which are designed via the universal approximation theorem [28].

In real applications of chaos synchronization, the state vectors of drive-response systems are not available for measurement, except the outputs of drive-response systems. Thus, designing a synchronization scheme based on an output-feedback controller (i.e., an observer-based controller) is required. Based on state observer, some adaptive control systems were designed in [29–32]. These systems involve strictly positive real (SPR) concept on the observation-error dynamics. The dynamics of the observation errors, which are originally not SPR, are augmented by an appropriate low-pass filter designed to meet the SPR concept.

On the other hand, most of the above works are only valid for chaotic systems without dynamical disturbances and input nonlinearity. However, in practice, the chaotic systems are inevitably affected by uncertain dynamical disturbances. The existence of these disturbances can generally lead to the synchronization failure and cause undesirable results. How to enhance the disturbance compensation or attenuation is of great significance [33, 34]. Besides, owing to the physical limitations, the practical implementations of the control systems are frequently exposed to input nonlinearities (backlash, dead-zone, and saturation). It has been shown that these input nonlinearities can cause a serious degradation of the system performances and in a worst-case system failure. So, the design of a controller for chaos synchronization by considering of the external disturbances and input nonlinearities is of significant importance [31–39]. To effectively deal with these problems, the control schemes have been generally designed in a variable-structure control framework.

Motivated by the above discussions, in this paper, we aim at addressing the problem of projective lag-synchronization for a class of uncertain chaotic systems subject to uncertain external dynamical disturbances and input nonlinearities (sector nonlinearities with dead-zone). This synchronization can be realized through an appropriate fuzzy adaptive variable-structure controller based on a state observer. Compared with the previous works on the chaos synchronization and control [8–14, 16–20, 31–39], the main contributions of this paper are the following:

(i) A novel projective lag-synchronization system based on fuzzy adaptive variable- structure output-feedback control is designed for unknown perturbed chaotic systems containing dead-zone nonlinearity.

(ii) The model of the chaotic drive-response system is assumed to be completely different, unknown (except its relative degree), subject to dynamical disturbances, with input dead-zone and sector nonlinearities, and immeasurable states. Besides, its dynamics should not satisfy the SPR property. To authors' best knowledge, such a class of chaotic (drive-response) systems with all these properties has not been previously considered in the open synchronization literature.

(iii) Unlike in [40–46], by using the SPR property together with Lyapunov theory, the stability of the resultant closed-loop system is carefully established. Recall that many previous works requiring the SPR property, for example, [40–46], have not been derived rigorously in mathematics, as stated in [47].

(iv) By designing a linear observer to estimate the lag-synchronization errors, only the outputs of the response-drive system are assumed to be measurable in this synchronization scheme.

(v) The designed fuzzy adaptive control is very simple and has only two adaptive parameters. So, this controller is of practical significant importance.

2. System Description and Problem Formulation

Consider the following class of drive-response chaotic systems:

$$\begin{aligned} y_x^{(n)} &= F_d(x) + D_d(t,x) \\ y_z^{(n)} &= F_r(z) + u + D_r(t,z) \end{aligned} \tag{1}$$

or equivalently of the form

$$\begin{aligned} \dot{x} &= Ax + B\left[F_d(x) + D_d(t,x)\right] \\ \dot{z} &= Az + B\left[F_r(z) + u + D_r(t,z)\right] \end{aligned} \tag{2}$$

with $A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, where $x = [x_1, \ldots, x_n]^T \in R^n$ and $z = [z_1, \ldots, z_n]^T \in R^n$ are the state vectors of the drive and response systems, respectively. $F_d(x)$ and $F_r(z)$ are unknown nonlinear smooth functions and $u = \varphi(v)$ is the input nonlinearity, with v being the control input which will be designed later. $D_d(t,x)$ and $D_r(t,z)$ are the external disturbances of the drive and response systems, respectively.

The input nonlinearity $u = \varphi(v)$ under consideration is given by [48–50]

$$\varphi(v) = \begin{cases} \varphi_+(v)(v - v_+), & v > v_+ \\ 0, & -v_- \le v \le v_+ \\ \varphi_-(v)(v + v_-), & v < -v_- \end{cases} \tag{3}$$

with $\varphi_+(v) > 0$ and $\varphi_-(v) > 0$ being nonlinear smooth functions of v, $v_+ > 0$ and $v_- > 0$. Note that this model contains both sector nonlinearity and dead-zone. The nonlinearity $\varphi(v)$ also has the following features:

$$\begin{aligned} (v - v_+)\varphi(v) &\ge m_+^*(v - v_+)^2, \quad v > v_+ \\ (v + v_-)\varphi(v) &\ge m_-^*(v + v_-)^2, \quad v < -v_-, \end{aligned} \tag{4}$$

with m_+^* and m_-^* being so-called "*the gain reduction tolerances*" [48–50].

Design Objective. Determine an output-feedback control law v to achieve a projective lag-synchronization between the drive system and the response one, while ensuring that all involved signals in the closed-loop system remain bounded.

To facilitate the control system design, the following usual assumptions are considered and will be used in the subsequent developments.

Assumption 1. The state vectors of the drive and response systems are not measurable, except the system outputs (i.e., except x_1 and z_1).

Assumption 2.

(i) The nonlinear functions $\varphi_+(v)$ and $\varphi_-(v)$ are unknown.

(ii) But, the constants v_+, v_-, m_+^*, and m_-^* are assumed to be known.

Assumption 3. The external disturbances, $D_d(t, x)$ and $D_r(t, z)$, are bounded, respectively, by

$$\begin{aligned}|D_d(t, x)| &\le c_d \\ |D_r(t, z)| &\le c_r,\end{aligned} \tag{5}$$

where c_d and c_r are some unknown positive constants.

Definition 4. The drive and response systems (2) are *projective lag-synchronized* if there exists a scaling factor λ such that $e = z - \lambda x(t - \tau) \to 0$ as $t \to \infty$, where $\tau > 0$ is a constant propagation delay or transmission delay. This means that the transmitted signal is received τ *time* late after it was sent. The value of τ depends on the channel or the distance between drive and response system.

Remark 5. From Definition 4, it is easy to see that, for $\tau = 0$, the complete synchronization, antisynchronization, and projective synchronization are the special cases when the scaling factor takes the values $\lambda = +1$, $\lambda = -1$, and $\lambda \ne 1$ and -1, respectively. And when $\lambda = 1$, one obtains the lag-synchronization.

From (2) and Definition 4, one can write the dynamics of the lag-synchronization error as

$$\begin{aligned}\dot{e} = \dot{z} - \lambda \dot{x}(t - \tau) &= Ae + B[-\lambda F_d(x_\tau) - \lambda D_d(t, x_\tau) \\ &\quad + F_r(z) + u + D_r(t, z)] = Ae + B[F_r(z) + u + P_1],\end{aligned} \tag{6}$$

where $x_\tau = x(t - \tau)$ and

$$P_1 = D_r(t, z) - \lambda F_d(x_\tau) - \lambda D_d(t, x_\tau). \tag{7}$$

Note that one can easily show the existence of a constant $c_1 > 0$ such as $|P_1| \le c_1$, for the following reasons: x evolves in a compact set (an intrinsic property of the (noncontrolled) chaotic systems), also the delayed state x_τ is bounded and the external disturbances, $D_d(t, x)$ and $D_r(t, z)$, are already assumed to be bounded, and finally the function $F_d(x_\tau)$ is smooth and with a bounded argument.

Since $F_r(z)$ is unknown and the vector e is immeasurable, in this paper, one will use

(1) a fuzzy adaptive system to approximate the uncertain functions,

(2) an observer to estimate the projective lag-synchronization error e.

3. Controller Design for Projective Lag-Synchronization

This section proposes a fuzzy adaptive output-feedback controller for lag-projective synchronization of the drive-response system (2) using Lyapunov stability theory. The proposed synchronization scheme is shown in Figure 1.

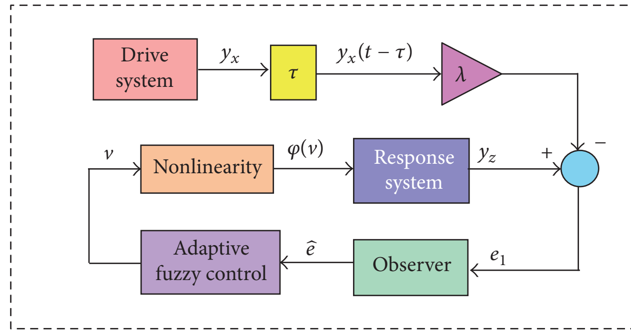

Figure 1: Projective lag-synchronization scheme.

One can rewrite the dynamics of the lag-synchronization errors as follows:

$$\begin{aligned}\dot{e} &= Ae + B[F_r(e + \lambda x(t - \tau)) + u + P_1] \\ e_1 &= Ce,\end{aligned} \tag{8}$$

where $C = [1\ 0\ \cdots\ 0]$. Note that the pair (C, A) is observable.

Remark 6.

(1) The controllability property of the pair (A, B) guarantees the existence of a feedback gain vector, K_c, so that the characteristic polynomial of $A - BK_c^T$ is strictly Hurwitz.

(2) The observability property of the pair (C, A) ensures the existence of an observer gain vector, K_o, so that the characteristic polynomial of $A - K_oC$ is strictly Hurwitz.

According to fuzzy approximation theorem [28], the unknown function $F_r(e + \lambda x(t - \tau))$ can be optimally approximated by a linearly parameterized fuzzy system, as follows [47]:

$$F_r(e + \lambda x(t - \tau)) = \theta^{*T}\psi(e) + \varepsilon(e, x(t - \tau)) \tag{9}$$

with $\psi(e)$ being the vector of FBFs (which are assumed to be designed a priori), $\varepsilon(e, x(t-\tau))$ being the fuzzy approximation error, and θ^* being the optimal value of the adjustable parameter vector of the fuzzy system (9) which is defined as

$$\theta^* = \arg\min_{\theta}\left[\sup_{e\in\Omega_e}\left|F_r(e + \lambda x(t - \tau)) - \theta^T\psi(e)\right|\right]. \tag{10}$$

According to [28], the fuzzy approximation error $\varepsilon(e, x(t-\tau))$ is bounded.

Then, (8) becomes

$$\begin{aligned}\dot{e} &= Ae + B[\theta^{*T}\psi(e) + u + P_2] \\ e_1 &= Ce,\end{aligned} \tag{11}$$

where $P_2 = P_1 + \varepsilon(e, x(t - \tau))$.

Since the lag-synchronization-error vector e is not available for measurement, one designs the following linear observer to estimate it:

$$\begin{aligned}\dot{\hat{e}} &= A_c\hat{e} + K_o\tilde{e}_1 \\ \hat{e}_1 &= C\hat{e},\end{aligned} \tag{12}$$

where $\hat{e}$ is the estimate of e, $K_o = [k_{o1}, \ldots, k_{on}]^T \in R^n$ is the gains vector of observer, $A_c = A - BK_c^T$, and $K_c = [k_{c1}, \ldots, k_{cn}]^T \in R^n$ is the feedback gain vector.

Now, one defines the observation-error vector as $\tilde{e} = [\tilde{e}_1, \ldots, \tilde{e}_n]^T = e - \hat{e}$. From (12) and (11), the dynamics of this observation error can be obtained as follows:

$$\begin{aligned}\dot{\tilde{e}} &= A_o\tilde{e} + B\left[\theta^{*T}\psi(e) + \varphi(v) + P_3\right] \\ \tilde{e}_1 &= C\tilde{e}\end{aligned} \tag{13}$$

with $A_o = A - K_oC$ and

$$P_3 = P_2 + K_c^T\hat{e}. \tag{14}$$

Then, we can rewrite (13) using the time-frequency (mixed) notation as follows [51, 52]:

$$\tilde{e}_1 = H(s)\left[\theta^{*T}\psi(e) + \varphi(v) + P_3\right], \tag{15}$$

where s is the Laplace variable and $H(s) = C(SI - A_o)^{-1}B$ is the stable transfer function of (13). It is worth noting that this mixed notation is very valuable in the adaptive control literature [51–56]. It also refers to the convolution between the inverse Laplace transform $H(s)$ and the term $\theta^{*T}\psi(e) + \varphi(v) + P_3$.

Since $H(s)$ is not SPR, one introduces a low-pass filter $T(s)$ such that $\overline{H}(s) = H(s)T^{-1}(s)$ becomes SPR:

$$\begin{aligned}&\tilde{e}_1 \\ &= \overline{H}(s)\left[\theta^{*T}T(s)[\psi(e)] + T(s)[\varphi(v)] + T(s)[P_3]\right] \\ &= \overline{H}(s)\left[\theta^{*T}\psi(\hat{e}) + \varphi(v) + P_4\right]\end{aligned} \tag{16}$$

with

$$\begin{aligned}P_4 = {}&\theta^{*T}T(s)[\psi(e)] + T(s)[\varphi(v)] + T(s)[P_3] \\ &- \theta^{*T}\psi(\hat{e}) - \varphi(v).\end{aligned} \tag{17}$$

Remark 7. $H(s)$ is SPR, with $s = \sigma + j\omega$ if the following conditions are satisfied [57]:

(a) When s is real, $H(s)$ is real.

(b) The poles of $H(s)$ are not in the right half-plane.

(c) For any real ω, the real part of $H(j\omega)$ is positive; that is, $\mathrm{Re}[H(j\omega)] \geq 0$.

Assumption 8. One assumes that $|P_4| \leq k_{p0}^* + k_{p1}^*|v| + k_{p2}^*|T(s)[v]| + k_{p3}^*|T(s)[K_c^T\hat{e}]| = K_p^{*T}W$, with $K_p^{*T} = [k_{p0}^*, k_{p1}^*, k_{p2}^*, k_{p3}^*]$ being an unknown positive vector, and $W^T = [1, |v|, |T(s)[v]|, |T(s)[K_c^T\hat{e}]|]$.

Let us define a novel error e_{m1}, called *the modified error*, as follows:

$$e_{m1} = \tilde{e}_1 + e_{a1} \tag{18}$$

with e_{a1} being *the auxiliary error*. Its dynamics are given by

$$e_{a1} = \overline{H}(s)\left[-K_p^TW\tanh\left(\frac{K_p^TWe_{m1}}{\varepsilon}\right)\right], \tag{19}$$

where K_p is the estimate of the unknown vector K_p^* and $\varepsilon > 0$ is a small design constant. $\tanh(\cdot)$ designates the usual hyperbolic tangent function.

From (16), (18), and (19), one can obtain

$$\begin{aligned}e_{m1} = \overline{H}(s)\Bigg[&\theta^{*T}\psi(\hat{e}) + \varphi(v) + P_4 \\ &- K_p^TW\tanh\left(\frac{K_p^TWe_{m1}}{\varepsilon}\right)\Bigg].\end{aligned} \tag{20}$$

The state-space presentation of (20) can be given by

$$\begin{aligned}\dot{e}_m &= \overline{A}_oe_m + \overline{B}\Bigg[\theta^{*T}\psi(\hat{e}) + \varphi(v) + P_4 \\ &\quad - K_p^TW\tanh\left(\frac{K_p^TWe_{m1}}{\varepsilon}\right)\Bigg] \\ e_{m1} &= \overline{C}e_m,\end{aligned} \tag{21}$$

where $e_m = [e_{m1}, \ldots, e_{mn}]^T$ and ($\overline{A}_o \in R^{n\times n}, \overline{B} \in R^{n\times 1}, \overline{C} \in R^{1\times n}$) is a minimal state realization of $\overline{H}(s) = H(s)T^{-1}(s) = \overline{C}^T(SI - \overline{A}_o)^{-1}\overline{B}$ and $\overline{C} = [1, 0, \ldots, 0]$.

Since $\overline{H}(s)$ is SPR, the following relation holds:

$$\begin{aligned}\overline{A}_oP + P\overline{A}_o &= -Q < 0 \\ P\overline{B} &= \overline{C}^T,\end{aligned} \tag{22}$$

where $P = P^T > 0$ and $Q = Q^T > 0$. Later, expressions (21) and (22) will be exploited in the stability analysis.

To achieve our objective, the control input can be determined as

$$v = \begin{cases}-\xi\rho\,\mathrm{sign}(e_{m1}) - v_-, & e_{m1} > 0 \\ 0, & e_{m1} = 0 \\ -\xi\rho\,\mathrm{sign}(e_{m1}) + v_+, & e_{m1} < 0\end{cases} \tag{23}$$

with $\xi > 1/\eta$, and $\eta = \min\{m_-^*, m_+^*\}$, where

$$\rho = w_2\|\psi(\hat{e})\| + w_1, \tag{24}$$

where w_1 is a design positive constant and w_2 is an adaptive parameter estimating the upper bound of $\|\theta^*\|$; that is, $w_2^* \geq \|\theta^*\|$.

The adaptive laws for the control law (23) are defined as

$$\begin{aligned}\dot{w}_2 &= -\gamma_w \sigma_w w_2 + \gamma_w \left|e_{m1}\right| \left\|\psi\left(\hat{e}\right)\right\|, \quad \text{with } w_2\left(0\right) > 0 \\ \dot{K}_p &= -\gamma_K \sigma_K K_p + \gamma_K \left|e_{m1}\right| W, \quad \text{with } K_p\left(0\right) > 0,\end{aligned} \tag{25}$$

where γ_K, σ_K, γ_w, and σ_w are strictly positive design parameters.

Theorem 9. *Consider the drive and response systems given by (2) (or (1)) under Assumptions 1–3 and 8. Then, the projective lag-synchronization is realized by using the fuzzy adaptive output-feedback controller (23)–(25) and observer (12).*

Proof of Theorem 9. Consider the following Lyapunov function:

$$V = \frac{1}{2} e_m^T P e_m + \frac{1}{2\gamma_K} \widetilde{K}_p^T \widetilde{K}_p + \frac{1}{2\gamma_w} \widetilde{w}_2^2, \tag{26}$$

where $\widetilde{K}_p = K_p - K_p^*$ and $\widetilde{w}_2 = w_2 - w_2^*$.

The time derivative of V is given as follows:

$$\dot{V} = \frac{1}{2} e_m^T P \dot{e}_m + \frac{1}{2} \dot{e}_m^T P e_m + \frac{1}{\gamma_K} \widetilde{K}_p^T \dot{K}_p + \frac{1}{\gamma_w} \widetilde{w}_2 \dot{w}_2. \tag{27}$$

Evaluating (27) along (21) and (25) and using Assumption 8, one gets

$$\begin{aligned}\dot{V} \leq{} & -\frac{1}{2} e_m^T Q e_m + e_{m1} \varphi\left(v\right) + w_2^* \left|e_{m1}\right| \left\|\psi\left(\hat{e}\right)\right\| \\ & + \left|e_{m1}\right| K_p^{*T} W - K_p^T W e_{m1} \tanh\left(\frac{K_p^T W e_{m1}}{\varepsilon}\right) \\ & + \frac{1}{\gamma_K} \widetilde{K}_p^T \dot{K}_p + \frac{1}{\gamma_w} \widetilde{w}_2 \dot{w}_2 \\ ={} & -\frac{1}{2} e_m^T Q e_m + e_{m1} \varphi\left(v\right) + \rho \left|e_{m1}\right| - w_1 \left|e_{m1}\right| \\ & + \left|e_{m1}\right| K_p^T W - K_p^T W \tanh\left(\frac{K_p^T W e_{m1}}{\varepsilon}\right) \\ & - \sigma_K \widetilde{K}_p^T K_p - \sigma_w \widetilde{w}_2 w_2.\end{aligned} \tag{28}$$

From (4) and (23), one can easily get the following expressions:

$$\begin{gathered} v < -v_- \quad \text{for } e_{m1} > 0 \Longrightarrow \\ \left(v + v_-\right) \varphi\left(v\right) \geq m_-^* \left(v + v_-\right)^2 \geq \eta \left(v + v_-\right)^2 \\ v > v_+ \quad \text{for } e_{m1} < 0 \Longrightarrow \\ \left(v - v_+\right) \varphi\left(v\right) \geq m_+^* \left(v - v_+\right)^2 \geq \eta \left(v - v_+\right)^2. \end{gathered} \tag{29}$$

Considering (23) again, one can establish

$$\begin{aligned} & e_{m1} > 0 \Longrightarrow \\ & \left(v + v_-\right) \varphi\left(v\right) = -\xi \rho \operatorname{sign}\left(e_{m1}\right) \varphi\left(v\right) \\ & \qquad \geq \eta \xi^2 \rho^2 \left[\operatorname{sign}\left(e_{m1}\right)\right]^2 \\ & e_{m1} < 0 \Longrightarrow \\ & \left(v - v_+\right) \varphi\left(v\right) = -\xi \rho \operatorname{sign}\left(e_{m1}\right) \varphi\left(v\right) \\ & \qquad \geq \eta \xi^2 \rho^2 \left[\operatorname{sign}\left(e_{m1}\right)\right]^2. \end{aligned} \tag{30}$$

Using the fact that $e_{m1} \operatorname{sign}(e_{m1}) = |e_{m1}|$ and while $\rho > 0$, one gets for all e_{m1}

$$e_{m1} \varphi\left(v\right) \leq -\xi \eta \rho \left|e_{m1}\right|. \tag{31}$$

Substituting (31) into (28), one can obtain

$$\begin{aligned}\dot{V} \leq{} & -\frac{1}{2} e_m^T Q e_m - \left(\xi \eta - 1\right) \rho \left|e_{m1}\right| - w_1 \left|e_{m1}\right| \\ & + \left|e_{m1}\right| K_p^T W - K_p^T W e_{m1} \tanh\left(\frac{K_p^T W e_{m1}}{\varepsilon}\right) \\ & - \sigma_K \widetilde{K}_p^T K_p - \sigma_w \widetilde{w}_2 w_2.\end{aligned} \tag{32}$$

On the other hand, one can establish that

$$\begin{aligned} & -\sigma_K \widetilde{K}_p^T K_p \leq -\frac{\sigma_K}{2} \left\|\widetilde{K}_p\right\|^2 + \frac{\sigma_K}{2} \left\|K_p^*\right\|^2 \\ & -\sigma_w \widetilde{w}_2 w_2 \leq -\frac{\sigma_w}{2} \widetilde{w}_2 + \frac{\sigma_w}{2} w_2^{*2} \\ & \left|K_p^T W e_{m1}\right| - K_p^T W e_{m1} \tanh\left(\frac{K_p^T W e_{m1}}{\varepsilon}\right) \leq \bar{\varepsilon} \\ & \quad = 0.2783 \varepsilon. \end{aligned} \tag{33}$$

By exploiting (33), (32) becomes

$$\dot{V} \leq -\frac{1}{2} e_m^T Q e_m - \frac{\sigma_K}{2} \left\|\widetilde{K}_p\right\|^2 - \frac{\sigma_w}{2} \widetilde{w}_2^2 + \pi, \tag{34}$$

where $\pi = \bar{\varepsilon} + (\sigma_K/2)\|K_p^*\|^2 + (\sigma_w/2) w_2^{*2}$.

Let $\mu = \min\{\lambda_{\min}(Q)/\lambda_{\max}(P), \gamma_w \sigma_w, \gamma_K \sigma_K\}$; hence one can rewrite (34) as follows:

$$\dot{V} \leq -\mu V + \pi, \tag{35}$$

where $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ are the smallest and largest eigenvalues of the matrix X, respectively.

(35) can be expressed as follows:

$$\frac{d\left(V e^{\mu t}\right)}{dt} \leq \pi e^{\mu t}. \tag{36}$$

And integrating (36) over $[0,t]$ yields

$$0 \leq V(t) \leq \frac{\pi}{\mu} + \left(V(0) - \frac{\pi}{\mu}\right) e^{-\mu t}. \quad (37)$$

Therefore all signals of the closed-loop system are bounded.

From (26) and (37), one has

$$\|e_m\| \leq \left(\frac{2}{\lambda_{\min}(P)}\left(\frac{\pi}{\mu} + \left(V(0) - \frac{\pi}{\mu}\right)e^{-\mu t}\right)\right)^{1/2}, \quad (38)$$

where $V(0) = (1/2)e_m^T(0)Pe_m(0) + (1/2\gamma_K)\widetilde{K}_p^T(0)\widetilde{K}_p(0) + (1/2\gamma_w)w_2^2(0)$.

From (38), one can conclude on the asymptotic convergence of the solution e_m to the following bounded region:

$$\Omega_{e_m} = \left\{e_m \mid \|e_m\| \leq \left(\frac{2}{\lambda_{\min}(P)}\frac{\pi}{\mu}\right)^{1/2}\right\}. \quad (39)$$

From (18), (19), and (39), one can establish easily the convergence and the boundedness of e_{a1} and $\widetilde{e}_1$.

The proof of this theorem is now completed. □

Remark 10. If $v_+ = v_- = v_0$, expression (23) can be simply rewritten as

$$v = -\left(\xi w_2 \|\psi(\widehat{e})\| + \xi w_1 + v_0\right) \operatorname{sign}(e_{m1}). \quad (40)$$

In (40), the sign function, that is, $\operatorname{sign}(e_{m1})$, can cause the undesirable chattering phenomenon. In practice, the latter is generally replaced by an equivalent and smooth function (e.g., $\tanh(k_{s1}e_{m1})$):

$$v = -\left(\xi w_2 \|\psi(\widehat{e})\| + \xi w_1 + v_0\right) \tanh(k_{s1}e_{m1}) \quad (41)$$

with $k_{s1} > 0$ being a high constant value.

Remark 11. More importantly, the design of a lag-synchronization system based on output-feedback controller for a class of uncertain drive-response systems with input nonlinearities has a major interest in both theory and practice.

(a) Theoretical Interests. Compared to previous works [8–14, 16–18], our theoretical contributions are the following:

(1) Design of a projective lag-synchronization system by considering the ubiquitous input nonlinearities (i.e., sector nonlinearities and dead-zone), the uncertain dynamics of both models, and the immeasurability of the states of drive-response system is theoretically challenge. To the best of authors' knowledge, the projective lag-synchronization for this class of drive-response systems with all these features has rarely been studied in the literature.

(2) The proposed fuzzy adaptive output-feedback control requires the so-called SPR condition on the lag-synchronization errors. It should be noted that the design of an output-feedback controller dealing with input nonlinearities (particularly, sector nonlinearity and dead-zone) and by using a SPR approach is not theoretically simple. This is why in the literature there are few fundamental results dealing with this control problem.

(b) Practical Interests. The proposed synchronization approach has the following practical interests:

(1) The proposed projective lag-synchronization approach is characterized by one scalar transmitted signal. This feature is of practical significant importance.

(2) The effect of ubiquitous input nonlinearities (sector nonlinearities and dead-zone) has been taken into account in the stability analysis and the design of the control system. In practice, it is well known that the nonconsideration of the latter may lead to a serious degradation of the system's performances and even cause system instability.

(3) In particular, this projective lag-synchronization approach has also a prospective application in secure communication due to its safety against attack and unmasking.

4. Illustrative Simulation Examples

Three academic examples are provided in this section to validate the effectiveness of this proposed synchronization approach.

Example 1. Consider the projective lag-synchronization between chaotic Gyros system and Duffing oscillator.

The Drive System (Chaotic Gyros System) [58]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\alpha^2 \frac{(1-\cos(x_1))^2}{\sin^3(x_1)} - c_1 x_2 - c_2 x_2^2 \\ &\quad + (\beta + f\sin(\omega_x t))\sin(x_1) + D_d(t,x), \end{aligned} \quad (42)$$

where $= [x_1 \ \ x_2]^T$, $\alpha^2 = 100$, $c_1 = 0.5$, $c_2 = 0.05$, $\beta = 1$, $\omega_x = 2$, and $f = 35.5$. $D_d(t,x)$ is assumed to be a normally (Gaussian) distributed random signal with a variance = 0.5 and a mean = 0.5.

The Response System (Duffing Oscillator) [59]

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -p_1 z_2 - p_2 z_1 - p_3 z_1^3 + q\sin(\omega_z t) + u \\ &\quad + D_r(t,z), \end{aligned} \quad (43)$$

where $z = [z_1 \ \ z_2]^T$, $p_1 = 0.4$, $p_2 = -1.1$, $p_3 = 1$, $q = 2.1$, $\omega_z = 1.8$, and $D_r(t,z) = \sin(6t)$.

Then, this chaotic drive-response system can be rewritten as follows:

$$\begin{aligned} \dot{x} &= Ax + B\left(F_d(x) + D_d(t,x)\right) \\ y_x &= x_1 = Cx \\ \dot{z} &= Az + B\left(F_r(z) + u + D_r(t,z)\right), \\ y_z &= z_1 = Cz, \end{aligned} \tag{44}$$

where $A = \left[\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right]$, $B = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$, and $C^T = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$. $u = \varphi(v)$ is the input nonlinearity which is defined below, and v is the control input to be designed.

The input nonlinearities $\varphi(v)$ are assumed to be described by [34, 48]

$$u = \varphi(v) = \begin{cases} (v - 0.5)\left(1.5 - 0.3e^{0.3|\sin(v)|}\right) & v > 0.5 \\ 0 & -0.5 \le v \le 0.5 \\ (v + 0.5)\left(1.5 - 0.3e^{0.3|\sin(v)|}\right) & v < -0.5. \end{cases} \tag{45}$$

To estimate the synchronization error, the following linear observer is designed:

$$\begin{aligned} \dot{\hat{e}} &= A_c\hat{e} + K_o\left(y_x(t-\tau) - y_z - \hat{e}_1\right) \\ \hat{e}_1 &= C\hat{e} \end{aligned} \tag{46}$$

with $\hat{e} = [\hat{e}_1, \hat{e}_2]^T$ being the estimate of $e = [e_1, e_2]^T$, $K_o = [2\alpha, \alpha^2]^T$ being the observer gain vector with $\alpha = 80$, $A_c = A - BK_c^T$, and $K_c = [90, 60]^T$.

Based on Theorem 9 and Remark 10, the control for system (44) can be designed as (40) or (41) with adaptive laws (25). Its associated design parameters are chosen as follows: $\tau = 0.5$ sec, $\lambda = 1$, $w_1 = 100$, $\varepsilon = 0.2$, $\gamma_w = 100$, $\sigma_w = 0.001$, $\gamma_k = 100$, and $\sigma_k = 0.001$. For each variable of the entries of the designed fuzzy system, as in [47, 60], one defines three membership functions (one triangular and two trapezoidal) uniformly distributed on the following intervals: $[-2 \;\; 2]$ for $\hat{e}_1$ and $[-2 \;\; 2]$ for $\hat{e}_2$.

One selects the SPR filter $T(s)$ so that $\overline{H}(s) = H(s)T^{-1}(s) = (1/(s^2 + 160s + 6400))T^{-1}(s)$ is SPR, as follows:

$$T(s) = \frac{1}{0.3906s + 11.7721}. \tag{47}$$

From the expression of $\overline{H}(s)$, one can find that $\overline{A} = \left[\begin{smallmatrix} -2\alpha & 1 \\ -\alpha^2 & 0 \end{smallmatrix}\right]$, $\overline{B}^T = [0.3906 \;\; 11.7721]$, and $\overline{C}^T = [1 \;\; 0]$.

By choosing $Q_1 = \left[\begin{smallmatrix} 30 & 3 \\ 3 & 0.5 \end{smallmatrix}\right]$ and solving (22), one gets

$$P_1 = \begin{bmatrix} 10.0937 & -0.2500 \\ -0.2500 & 0.0083 \end{bmatrix}. \tag{48}$$

The initial conditions are chosen as $x(0) = [x_1(0), x_2(0)]^T = [-1, 1]^T$, $z(0) = [z_1(0), z_2(0)]^T = [0.5, 2]^T$, $w_2(0) = 10$, and $K_p(0) = [0.01, 0.01, 0.01, 0.01]^T$.

Note that, because $v_+ = v_- = v_0 = 0.5$, the variable-structure controller (23) can be directly replaced by (40). Two cases are considered to show the validity of the proposed controller.

(a) Simulation by Using the Discontinuous Controller (40). Figure 2 shows that the proposed controller performs well. In fact, one can see from Figures 2(a) and 2(b) that the states of response system (z_1, z_2) effectively track that of the drive system $(\lambda x_1(t-\tau), \lambda x_2(t-\tau))$, despite the presence of the immeasurable states, uncertain dynamics, dead-zone at the input, and external disturbances. From Figure 2(c), it is clear also that the estimates of the synchronization errors are bounded and asymptotically converge towards small values. The corresponding control signal is bounded and not smooth in Figure 2(d).

(b) Simulation by Using the Smooth Controller (41). Figure 3 provides the simulation results. From Figures 3(a) and 3(b), one can observe that the states of the response system (z_1, z_2) effectively follow the corresponding desired trajectories $(\lambda x_1(t-\tau), \lambda x_2(t-\tau))$. From Figure 3(c), one can see that the estimates of the synchronization errors are well-bounded and converge to a small value. In Figure 3(d), the control signal is smooth, bounded, and admissible.

Example 2. Now, we will consider the projective lag-synchronization between two uncertain similar chaotic systems of the third order.

The Drive System (Genesio Chaotic System) [61]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -6x_1 - 2.92x_2 - 1.2x_3 + x_1^2 + D_d(t,x), \end{aligned} \tag{49}$$

where $x = [x_1, x_2, x_3]^T$ and $D_d(t,x)$ is assumed to be a normally (Gaussian) distributed random signal with a variance = 0.5 and a mean = 0.5.

The Response System (Genesio Chaotic System) [61]

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= -6z_1 - 2.92z_2 - 1.2z_3 + z_1^2 + u + D_r(t,z), \end{aligned} \tag{50}$$

where $D_r(t,z) = \sin(6t)$ and $z = [z_1, z_2, z_3]^T$ is the state vector of the response system.

Then, this chaotic drive-response system can be rewritten as follows:

$$\begin{aligned} \dot{x} &= Ax + B\left(F_d(x) + D_d(t,x)\right), \\ y_x &= x_1 = Cx \\ \dot{z} &= Az + B\left(F_r(z) + u + D_r(t,z)\right), \\ y_z &= z_1 = Cz, \end{aligned} \tag{51}$$

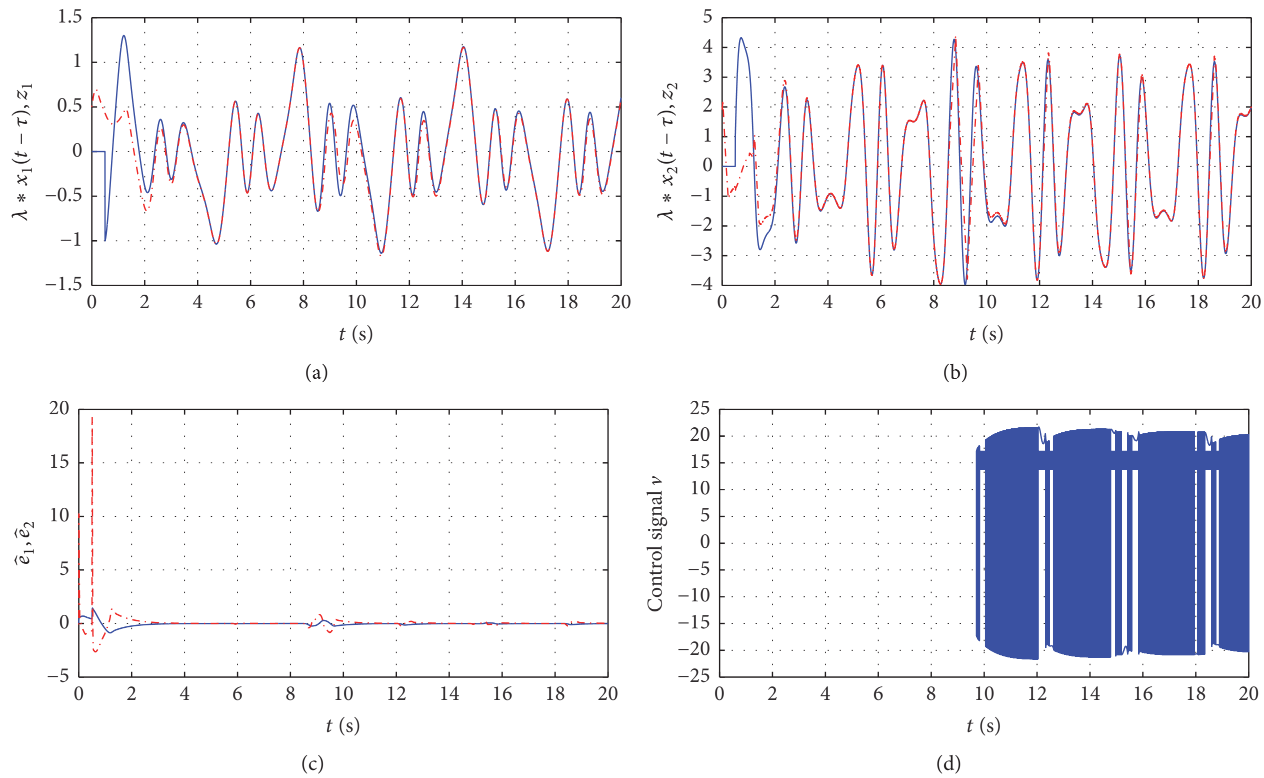

Figure 2: Simulation results (for Example 1, in case 1): (a) states: $\lambda x_1(t-\tau)$ (solid line) and z_1 (dash-dot line). (b) States: $\lambda x_2(t-\tau)$ (solid line) and z_2 (dash-dot line). (c) Estimates of the synchronization errors $\hat{e}_1$ (solid line) and $\hat{e}_2$ (dash-dot line). (d) Control signal v.

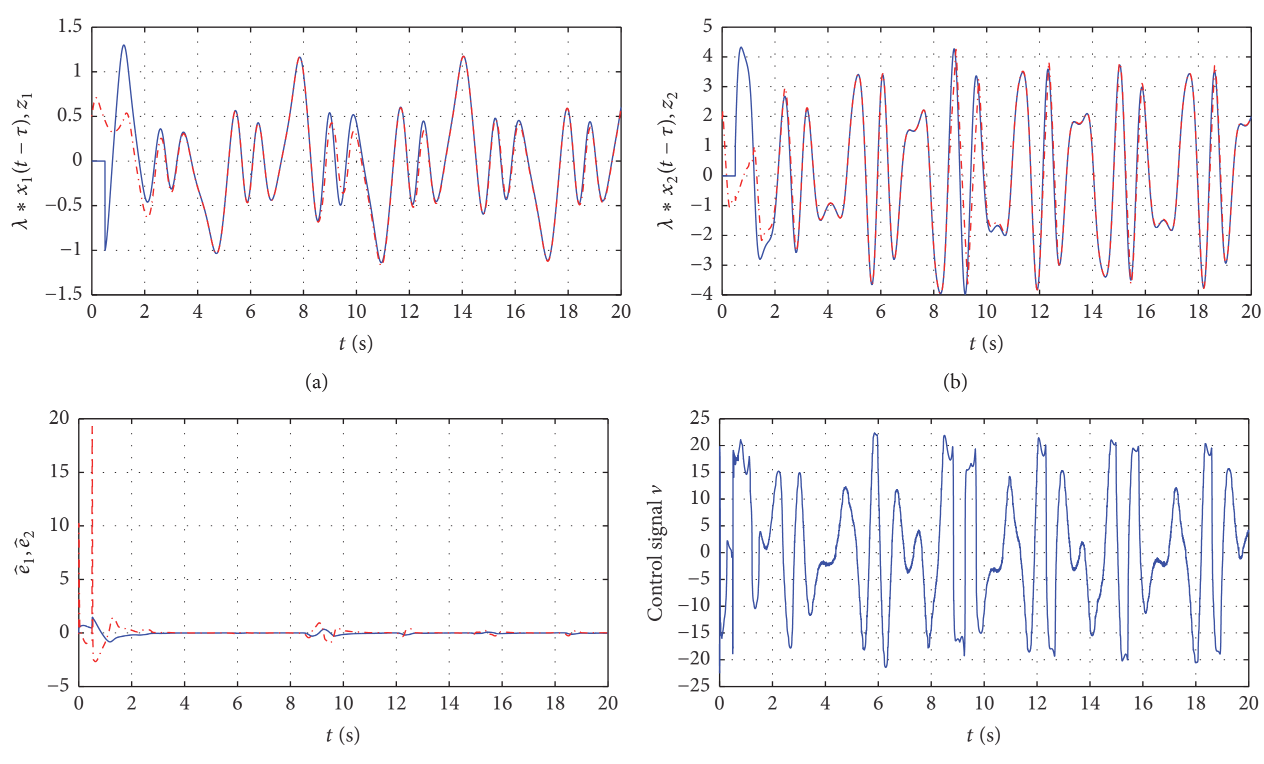

Figure 3: Simulation results (for Example 1, in case 2): (a) states: $\lambda x_1(t-\tau)$ (solid line) and z_1 (dash-dot line). (b) States: $\lambda x_2(t-\tau)$ (solid line) and z_2 (dash-dot line). (c) Estimates of the synchronization errors $\hat{e}_1$ (solid line) and $\hat{e}_2$ (dash-dot line). (d) Control signal v.

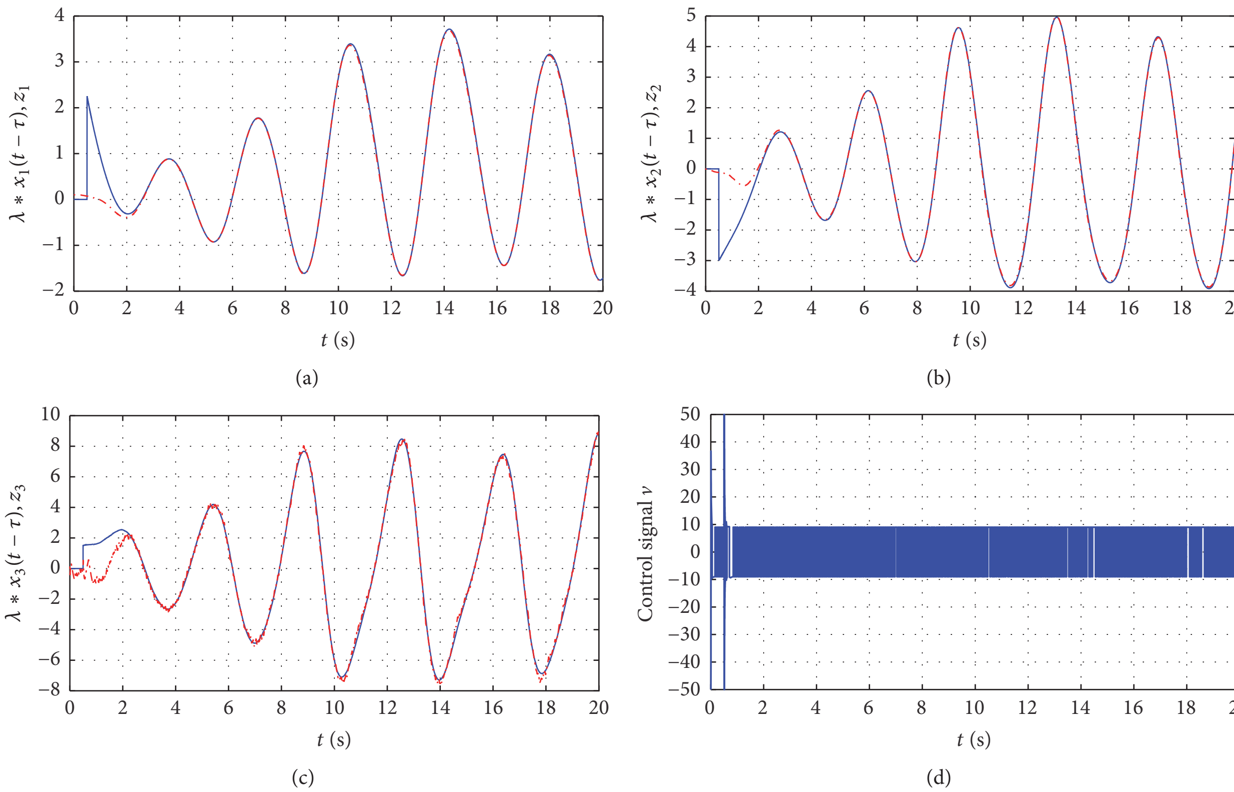

Figure 4: Simulation results (for Example 2): (a) states: $\lambda x_1(t-\tau)$ (solid line) and z_1 (dash-dot line). (b) States: $\lambda x_2(t-\tau)$ (solid line) and z_2 (dash-dot line). (c) States: $\lambda x_3(t-\tau)$ (solid line) and z_3 (dash-dot line). (d) Control signal v.

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and $C = [1\ \ 0\ \ 0]$. $u = \varphi(v)$ is the input nonlinearity which is defined below, and v is the control input to be designed.

The input nonlinearities $\varphi(v)$ are assumed to be described by [34, 48]

$$u = \varphi(v) = \begin{cases} (v-1)\left(1.5 - 0.3e^{0.3|\sin(v)|}\right) & v > 1 \\ 0 & -1 \le v \le 1 \\ (v+1)\left(1.5 - 0.3e^{0.3|\sin(v)|}\right) & v < -1. \end{cases} \tag{52}$$

To estimate the synchronization error, the following linear observer is designed:

$$\begin{aligned} \dot{\hat{e}} &= A_c\hat{e} + K_o\left(y_x(t-\tau) - y_z - \hat{e}_1\right) \\ \hat{e}_1 &= C\hat{e} \end{aligned} \tag{53}$$

with $\hat{e} = [\hat{e}_1, \hat{e}_2, \hat{e}_3]^T$ being the estimate of $e = [e_1, e_2, e_3]^T$, $K_o = [3\alpha, 3\alpha^2, \alpha^3]^T$ being the observer gain vector with $\alpha = 60$, $A_c = A - BK_c^T$, and $K_c = [64, 48, 12]^T$.

Based on Theorem 9 and Remark 10, the controller for system (51) can be designed as (40) with adaptive laws (25). Its associated design parameters are chosen as follows: $\tau = 0.5$ sec, $\lambda = 0.75$, $w_1 = 1$, $\varepsilon = 0.2$, $\gamma_w = 1000$, $\sigma_w = 0.01$, $\gamma_k = 100$, and $\sigma_k = 2$. For each input of the fuzzy system, as in [47, 60], one designs three membership functions (the central membership function is triangular and, however, the two others are trapezoidal) uniformly distributed on the following intervals: $[-5\ \ 5]$ for $\hat{e}_1$, $[-5\ \ 5]$ for $\hat{e}_2$, and $[-10\ \ 10]$ for $\hat{e}_3$.

We select the SPR filter $T(s)$ so that $\overline{H}(s) = H(s)T^{-1}(s) = (1/(s^3 + 180s^2 + 10800S + 216000))T^{-1}(s)$ is SPR, as follows: $T(s) = 1/(0.0003s^2 + 0.0120S + 0.2726)$.

From the expression of $\overline{H}(s)$, one can find that $\overline{A} = \begin{bmatrix} -3\alpha & 1 & 0 \\ -3\alpha^2 & 0 & 1 \\ -\alpha^3 & 0 & 0 \end{bmatrix}$, $\overline{B}^T = [0.0003\ \ 0.0120\ \ 0.2726]$, and $\overline{C}^T = [1\ \ 0\ \ 0]$.

By choosing $Q = \begin{bmatrix} 30 & 3 & 3 \\ 3 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ and solving (22), one gets

$$P = \begin{bmatrix} 14985 & -2 & -12 \\ -2 & 11 & -1 \\ -12 & -1 & 0.0001 \end{bmatrix}. \tag{54}$$

The initial conditions are chosen as $x(0) = [x_1(0), x_2(0), x_3(0)]^T = [3, -4, 2]^T$, $z(0) = [z_1(0), z_2(0), z_3(0)]^T = [0.1, 0.0, 0.1]^T$, $w_2(0) = 40$, and $K_p(0) = [0, 0, 0, 0]^T$.

The projective lag-synchronization response of system (51) is presented in Figure 4. It is obvious from the latter that the trajectories of response system (z_1, z_2, z_3) effectively track that of the drive system $(\lambda x_1(t-\tau), \lambda x_2(t-\tau), \lambda x_3(t-\tau))$, despite the presence of the immeasurable states of chaotic systems, uncertain dynamics, dead-zone at the input, and

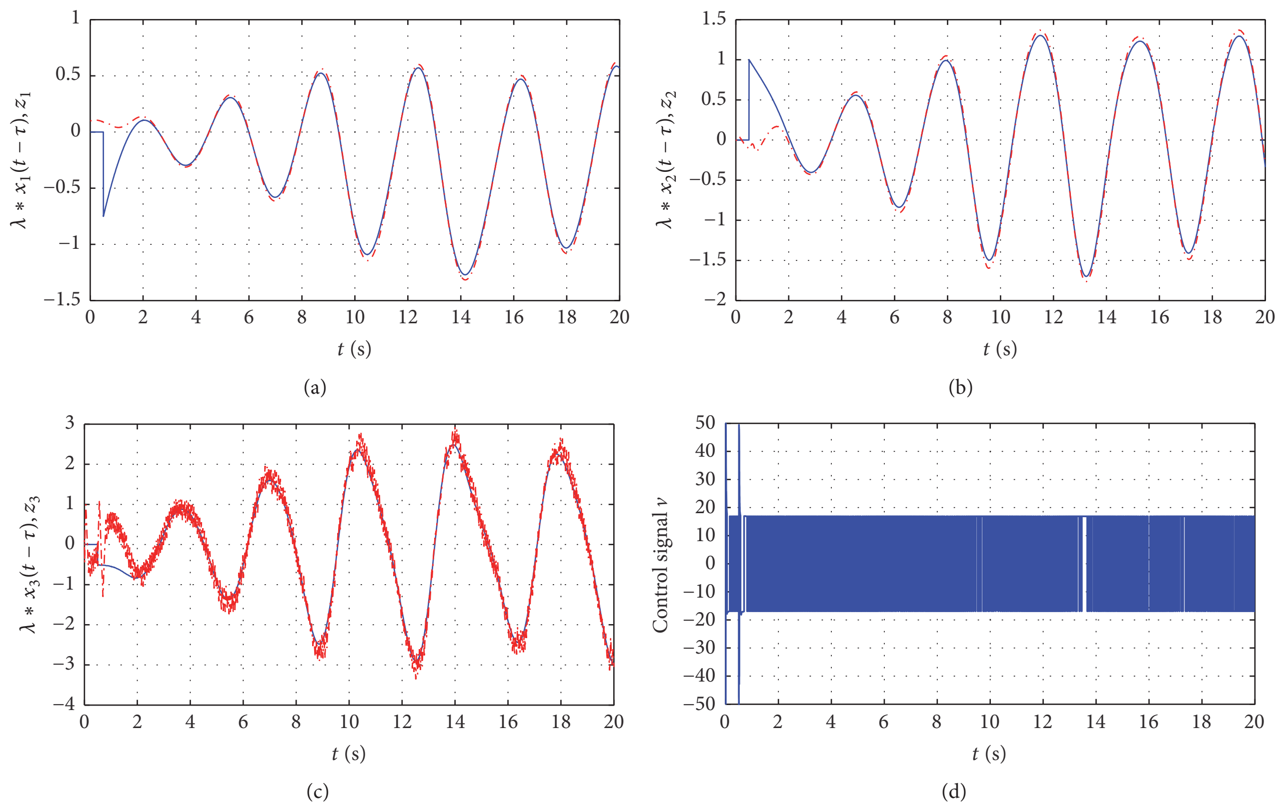

FIGURE 5: Simulation results (for Example 3): (a) states: $\lambda x_1(t-\tau)$ (solid line) and z_1 (dash-dot line). (b) States: $\lambda x_2(t-\tau)$ (solid line) and z_2 (dash-dot line). (c) States: $\lambda x_3(t-\tau)$ (solid line) and z_3 (dash-dot line). (d) Control signal v.

external disturbances. The corresponding control signal is bounded.

Example 3. We consider now the projective lag-synchronization between two different chaotic systems of the third order.

The Drive System (Genesio Chaotic System)

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -6x_1 - 2.92x_2 - 1.2x_3 + x_1^2 + D_d(t,x), \end{aligned} \tag{55}$$

where $x = [x_1, x_2, x_3]^T$ and $D_d(t,x)$ is taken as a normally (Gaussian) distributed random signal with a variance = 0.5 and a mean = 0.5.

The Response System (Arneodo Chaotic System) [62]

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= 5.5z_1 - 3.5z_2 - z_3 - z_1^3 + u + D_r(t,z), \end{aligned} \tag{56}$$

where $D_r(t,z) = \sin(6t)$ and $z = [z_1, z_2, z_3]^T$.

Note that this synchronization scheme is designed as that of the second example, except that the response system is selected as an Arneodo chaotic system and $\lambda = -0.25$. Therefore, the fuzzy system and the synchronization-error observer are designed as in Example 2, and the initial conditions and the design parameters are also selected as in Example 2.

Figure 5 provides the simulation results of this example, from which it can be clearly seen that the trajectories of response system (z_1, z_2, z_3) effectively track the trajectories of the drive system $(\lambda x_1(t-\tau), \lambda x_2(t-\tau), \lambda x_3(t-\tau))$, despite the presence of immeasurable states of chaotic systems, uncertain dynamics, input nonlinearities, and external disturbances.

5. Conclusion

The problem of adaptive fuzzy output-feedback control-based projective lag-synchronization for unknown drive-response (or master-slave) chaotic systems has been investigated in this paper. In the design process, the input nonlinearities (dead-zone together with sector nonlinearities) have been considered. To effectively handle the unknown functions in the drive-response system, fuzzy adaptive systems have been incorporated in the control system. To deal with the input nonlinearities, the proposed controller has been designed in a variable-structure framework. And to estimate the synchronization-error states, a simple linear observer has been constructed. Finally, three academic examples have been given to demonstrate the effectiveness of the proposed lag-synchronization approach. In our future work, the investigation for chaotic fractional-order drive-response systems subject to unavailable states and more nonsmooth input nonlinearities deserves further research.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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